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fields which have been investigated. It is the exception that something can be done by known methods, the rule is that it cannot yet be done.

When we study the literature of some of the older subjects we are sometimes surprised by the large number of known facts, but when we come to study the subjects themselves and ask independent questions we are generally surprised to learn that so few properties are known. Hence it seems very desirable that the advanced student, at least, should study subjects rather than the known facts in regard to these subjects. In this way a more accurate idea of the true state of knowledge can be obtained. Besides the knowledge of having discovered facts and relations which will enter into the structure of a growing science is the greatest source of pleasure that the student can obtain.

Cornell University, March 9, 1900.

AN ELEMENTARY DERIVATION OF THE SERIES FOR $\sin x$ and $\cos x$.

By H. C. WHITAKER, Ph. D., Philadelphia, Pa.

Assume $\sin x = x + a_3x^3 + a_5x^5 + \ldots\ldots\ldots\ldots\ldots\ldots(1)$,

$\cos x = 1 + a_2x^2 + a_4x^6 + \ldots\ldots\ldots\ldots\ldots\ldots(2)$.

Then $\sin y = y + a_3y^3 + a_5y^5 + \ldots\ldots\ldots\ldots\ldots\ldots(3)$,

$\cos y = 1 + a_2y^2 + a_4y^6 + \ldots\ldots\ldots\ldots\ldots\ldots(4)$.

Also $\sin(x+y) = x + y + a_3(x+y)^3 + a_5(x+y)^5 + \ldots\ldots\ldots\ldots(5)$,

$\cos(x+y) = 1 + a_2(x+y)^2 + a_4(x+y)^4 + \ldots\ldots\ldots\ldots(6)$.

But $\sin(x+y) = \sin x\cos y + \cos x\sin y \ldots\ldots\ldots\ldots(7)$,

$\cos(x+y) = \cos x\cos y - \sin x\sin y \ldots\ldots\ldots\ldots(8)$.

Hence the coefficients $a_2, a_3, a_4 \ldots\ldots$ can be determined by equating (5) to $(1)\times(4)+(2)\times(3)$, and by equating (6) to $(2)\times(4)-(1)\times(3)$. In the expansion, in each case, pick out only the terms containing the first power of y. In (5) and (7), the terms are

$$y + 3a_3x^2y + 5a_5x^4y +$$
$$y + a_2x^2y + a_4x^4y +$$

In (6) and (8), the terms are

$$2a_2xy + 4a_4x^3y + 6a_6x^5y +$$
$$-xy - a_8x^3y - a_5x^5y +$$

Equating coefficients of like powers, $2a_2=-1$, $3a_3=a_2$, $4a_4=-a_3$, and $5a_5=a_4$.

Solving for a_2, a_3, a_4, a_5, and substituting in (1) and (2),

$$\sin x=x-\frac{x^3}{3\,!}+\frac{x^5}{5\,!}-$$

$$\cos x=1-\frac{x^2}{2\,!}+\frac{x^4}{4\,!}-$$

INTEGRATION OF ELLIPTIC INTEGRALS.

By G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.

[Continued from March Number.]

Writing $s+s^{-1}$ for $\cos 2\varphi$ in (B), (C), (D), (E) we get

$$2(1+e^2-es-es^{-1})^{-\frac{1}{2}}=A_0+A_1(s+s^{-1})+A_2(s^2+s^{-2})+A_3(s^3+s^{-3})$$
$$+\ldots\ldots+A_n(s^n+s^{-n})+\quad\ldots\ldots\ldots(37).$$

$$2(1+e^2-es-es^{-1})^{-\frac{3}{2}}=B_0+B_1(s+s^{-1})+B_2(s^2+s^{-2})+B_3(s^3+s^{-3})$$
$$+\ldots\ldots+B_n(s^n+s^{-n})+\quad\ldots\ldots\ldots(38).$$

$$2(1+e^2-es-es^{-1})^{-\frac{5}{2}}=C_0+C_1(s+s^{-1})+C_2(s^2+s^{-2})+C_3(s^3+s^{-3})$$
$$+\ldots\ldots+C_n(s^n+s^{-n})+\quad\ldots\ldots\ldots(39).$$

$$2(1+e^2-es-es^{-1})^{-\frac{7}{2}}=D_0+D_1(s+s^{-1})+D_2(s^2+s^{-2})+D_3(s^3+s^{-3})$$
$$+\ldots\ldots+D_n(s^n+s^{-n})+\quad\ldots\ldots\ldots(40).$$

Differentiating (37), (38), and (39) we get

$$e(1-s^{-2})(1+e^2-es-es^{-1})^{-\frac{3}{2}}=A_1(1-s^{-2})+2A_2(s-s^{-3})+3A_3(s^2-s^{-4})$$
$$+\ldots\ldots+nA_n(s^{n-1}-s^{-n-1})+\ldots\ldots(41).$$

$$3e(1-s^{-2})(1+e^2-es-es^{-1})^{-\frac{5}{2}}=B_1(1-s^{-2})+2B_2(s-s^{-3})+3B_3(s^2-s^{-4})$$
$$+\ldots\ldots+nB_n(s^{n-1}-s^{-n-1})+\ldots\ldots(42).$$

$$5e(1-s^{-2})(1+e^2-es-es^{-1})^{-\frac{7}{2}}=C_1(1-s^{-2})+2C_2(s-s^{-3})+3C_3(s^2-s^{-4})$$
$$+\ldots\ldots+nC_n(s^{n-1}-s^{-n-1})+\ldots\ldots(43).$$